Assignment for class 12

General direction for candidates: Notes provided must be copied in Maths copy and then homework should be done in the same copy.

Determinant (Properties of determinants)

- Helps in simplifying its evaluation by obtaining maximum number of zeros in a row or a column
- These properties are true for determinants of any order.

Note : (For proof of properties refer to the video link in you tube provided by school)

Property 1 : If each element of a row (or column) of a determinant is zero, then its value is zero.

Property 2: If each element on one side of the principal diagonal of a determinant is zero then the the value of determinant is the product of the principal diagonal elements.

Property3: The value of determinant remains unchanged if its rows and columns are interchanged.

Note :It follows from the above property that if A is a square matrix, then

det (A^T)=det (A) (where A^T is transpose of A)

Property 4: If any two Rows (or columns) of a determinant are interchanged , then the value of the determinant changes by minus sign only.

Note: It follows from above property that if any row (or column)of a determinant be be passed over m rows (or column) , then the resulting determinant Δ = $(-1)^m$Δ.

Property 5: If two parallel lines (rows or column) of a determinant are identical (all corresponding elements are same), then the value of determinant is zero**.**

Property 6: If each of element of a row (or a column) of a determinant is multiplied by the same number **k** ,then the value of the new determinant is **k** times the value of the original determinant.

Note:

- **By the above property , we can take out any common factor from any one row or any one column of a given determinant.**
- **If *k*A is matrix obtained by multiplying each element of matrix A with number *k* therefore , if A is a square matrix of order n, then $|kA|=k^n\ |A|$ (Because common factor *k* will be taken out from each of n -rows or n -columns)**
  thus if each element of a determinant Δ is multiplied by the same number *k* and Δ_1 is the new determinant then
  $\Delta_1 = k\,\Delta$ if order of Δ =1
  $\Delta_1 = k^2\,\Delta$ if order of Δ =2
  $\Delta_1 = k^3\Delta$ if order of Δ=3

NOTE $*$ **(Also mentioned in assignment 2...k *is no. multiplied to each element of matrix A)***
It follows from property 5 & property 6 :
If two parallel lines (rows and column) of a determinant are such that the elements of one line are equi-multiples of the elements of the other line , then the value of determinant is zero)

Property 7: If each element of a row (or column) of a determinant consist of two or more terms , then the determinant can be expressed as the sum of two or more determinants whose other rows (or columns) are not altered.

Property 8: If to each element of a row(or a column) of a determinant be added the equi-multiples of the corresponding elements of one or more rows (or columns), the value of the determinants remain unchanged.

Property 9: The sum of the product of elements of any row (or column)with the cofactor of the corresponding elements of some other row (or column) is zero .

Elementary Operations: If Δ be Determinant of order n , where n ≥2: R_1, R_2.... its first row, second row,.....and C_1, C_2,.....denote its first column,second column,....respectively.

- The operations of interchanging the ith row and jth column of Δ is denoted by $R_i \leftrightarrow R_j$ and for column $C_i \leftrightarrow C_j$
- Operation of multiplying each element of the ith row of determinant by number k will be denoted by $R_i \rightarrow kR_i$ and for column $C_i \rightarrow kC_i$

  Note: If we apply $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$, then we multiply determinant by $\frac{1}{k}$ in the same step

- The operation of adding to each element of the ith row of Δ, k times the corresponding elements of the jth row (j≠i) will be denoted by $R_i \rightarrow R_i + kR_j$, and for column $C_i \rightarrow C_i + kC_j$

Exercise 4.2. Q3i) Evaluate without expanding $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Operating $R_1 \rightarrow R_1 - 6R_3$ **(**Property 8), we get

$$= \begin{vmatrix} 102-6\times17 & 18-6\times3 & 36-6\times6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0$$ (By property 1)

Homework Exercise 4.2 Q3ii) Q4.i) ,iii)

Q5i) Given A is Square matrix of order 2, |A|= -5, Find |3A|

Since $|KA|=K^n|A|$ **(refer to Property 6)**

$|3A|= 3^2|A|=9(-5)= -45$

Homework : Exercise 4.2 Q5 (iii),(v) & (vi) Q7 (Hint for Q7: $|A^m|=|A|^m$, if A is Square matrix &m$\in$**N)**

Q8. Using properties of determinants solve for x:

Given $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

$\Delta=\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix}$

Applying $R_1 \rightarrow R_1+R_2+R_3$

$=\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix}$

Taking (3x+a) common from R_1

$=(3x+a)\begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix}$

Applying $C_1 \rightarrow C_1-C_2$

$=(3x+a)\begin{vmatrix} 1 & 0 & 1 \\ x & a & x \\ x & 0 & x+a \end{vmatrix}$

Applying $C_3=C_3-C_1$

$=(3x+a)\begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix}$

$= (3x+a)\left(1\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix}\right)= (3x+a)(a^2)$ (Expand along R_1)

therefore (3x+a)(a^2)=0 since Δ=0

x= $\frac{-a}{3}$

Homework : Q,11 i) & ii) Q12 i)&ii) ***(Note; S0lution of Q12iii) is there in the video*** **) ,29 iii)**

Q16. using properties of determinant prove the identities

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = (x-y)(y-z)(z-x)$$

= **By** applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & (y-x)(y+x) \\ 0 & z-x & (z-x)(z+x) \end{vmatrix}$$

$= (y-x)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$ Taking (y-x) common from R_2 & (z-x) common from R_3

= (y-x)(z-x) (1) $\begin{vmatrix} 1 & y+x \\ 1 & z+x \end{vmatrix}$ Expanding along R_1

=(y-x)(z-x(z+x-y-x) = (y-x)(z-x)(z-y)

= (-1)(x-y)(-1)(y-z)(z-x)

= (x-y)(y-z)(z-x)

Similarly solve $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$ as **Homework**

Homework Q17, Q.18ii) Q,22,

Q.20 $\begin{vmatrix} 1 & \alpha & \alpha^2+\beta\gamma \\ 1 & \beta & \beta^2+\gamma\alpha \\ 1 & \gamma & \lambda^2+\alpha\beta \end{vmatrix} = 2(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

By applying property 7

$$=\begin{vmatrix}1 & \alpha & \alpha^2\\1 & \beta & \beta^2\\1 & \gamma & \gamma^2\end{vmatrix}+\begin{vmatrix}1 & \alpha & \beta\gamma\\1 & \beta & \gamma\alpha\\1 & \gamma & \alpha\beta\end{vmatrix}$$ (now proceed as in Q16)

Q21, Using properties prove : $\begin{vmatrix}a^2+1 & ab & ac\\ab & b^2+1 & bc\\ac & bc & c^2+1\end{vmatrix}=\begin{vmatrix}a^2+1 & b^2 & c\\a^2 & b^2+1 & c\\a^2 & b^2 & c^2+1\end{vmatrix}$ **=1 +a² +b² +c²**

$$\begin{vmatrix}a^2+1 & ab & ac\\ab & b^2+1 & bc\\ac & bc & c^2+1\end{vmatrix}$$

Taking ***a*** common from R_1, ***b*** common from R_2 & ***c*** common from R_3

$$=abc\begin{vmatrix}\frac{a^2+1}{a} & b & c\\a & \frac{b^2+1}{b} & c\\a & b & \frac{c^2+1}{c}\end{vmatrix}$$ Property 6

= operate $C_1 \to aC_1$, $C_2 \to bC_2$, & $C_3 \to cC_3$ ie (Multiplying C_1 by a, C_2 by b, & C_3) by c we get

$$\frac{abc}{abc}\begin{vmatrix}a^2+1 & b^2 & c^2\\a^2 & b^2+1 & c^2\\a^2 & b^2 & c^2+1\end{vmatrix}=\begin{vmatrix}a^2+1 & b^2 & c^2\\a^2 & b^2+1 & c^2\\a^2 & b^2 & c^2+1\end{vmatrix}$$ proved

$$\begin{vmatrix}a^2+1 & b^2 & c^2\\a^2 & b^2+1 & c^2\\a^2 & b^2 & c^2+1\end{vmatrix}$$

By operation $C_1 \to C_1 + C_2 + C_3$

$$=\begin{vmatrix}1+a^2+b^2+c^2 & b^2 & c^2\\1+a^2+b^2+c^2 & b^2+1 & c^2\\1+a^2+b^2+c^2 & b^2 & c^2+1\end{vmatrix}$$

by taking $(1+a^2+b^2+c^2)$ common from C_1

$$= (1 + a^2 + b^2 + c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$$

by applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$== (1 + a^2 + b^2 + c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

by Expansion along C_1

$$=(1 + a^2 + b^2 + c^2) \left(1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\right)$$

$$=(1 + a^2 + b^2 + c^2)$$

Homework : Complete Q.20, and solve Q.30, Q25ii) ,Q31